

# **WELCOME TO THE 2019 SUMMER SEMINAR IN PHIL STAT**

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# The Statistics Wars



## **2. Role of Probability: performance or probabilism? (Frequentist vs. Bayesian)**

- Need new role: Probativism

# Simple significance tests (Fisher)

**p-value.** ...to test the conformity of the particular data under analysis with  $H_0$  in some respect:

...we find a function  $d(\mathbf{X})$  of the data, the **test statistic**, such that

- the larger the value of  $d(\mathbf{X})$  the more inconsistent are the data with  $H_0$ ;
- $d(\mathbf{X})$  has a known probability distribution when  $H_0$  is true.

...the p-value corresponding to any  $d(\mathbf{x})$  (or  $d_{obs}$ )

$$p = p(t) = \Pr(d(\mathbf{X}) \geq d(\mathbf{x}); H_0)$$

(Mayo and Cox 2006, 81; d for t, x for y)



# Testing Reasoning

- If even larger differences than  $d_{obs}$  occur fairly frequently under  $H_0$  (i.e., P-value is not small), there's scarcely evidence of incompatibility with  $H_0$
- Small P-value indicates *some* underlying discrepancy from  $H_0$  because **very probably you would have seen a less impressive** difference than  $d_{obs}$  were  $H_0$  true.

Popper ( $\mu > \mu_1$ ) is corroborated (at level .975) because *it may be presented as a failed attempt to falsify it statistically.*

FEV(i) (p. 149):  $x$  is evidence against  $H_0$  (evidence of a discrepancy from  $H_0$ ), if and only if, were a correct description of the mechanism generating  $x$ , then, with **high probability**, this would have resulted in a less discordant result than is exemplified by  $x$  (Mayo and Cox 2006, p. 82).

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FEV/SEV (i) (p. 149):  $x$  is evidence against  $H_0$  (evidence of a discrepancy from  $H_0$ ), if and only if, the P-value  $\Pr(d \geq d_0; H_0)$  probability is very low (equivalently,  $\Pr(d < d_0; H_0) = 1 - \text{P-value}$  is very high).

Small

Here's your severity: there's a high probability the results would have gone (more) in a direction against what you're taking to infer H, if H is false.

Pr (Evidence in disagreement with your inference to C against  $H_0$ ;  $H_0$ ) would be **high**

You get inference that accords with C  
Data indicate (or supply evidence for C)

and the warrant is this high probability



## Variations on Test T+ (new ex starts p. 29)

### Test (1)

1-sided normal testing

$H_0: \mu \leq 150$  vs.  $H_1: \mu > 150$  (Let  $\sigma = 10$ ,  $n = 100$ )

let significance level  $\alpha = .025$

Reject  $H_0$  whenever  $M \geq 150 + 2\sigma/\sqrt{n}$ :

$M \geq 152$

$M$  is the sample mean, its value is  $M_0$ .

$1SE = \sigma/\sqrt{n} = 1$

## **P-value:**

$$\Pr( M \geq 152; \mu = 150)$$

Turn M into a Z statistics

$$Z = (152 - 150)/ SE$$

$$(\text{observed} - \text{hypothesized})/SE$$

$$Z = 2$$

So want  $\Pr(Z \geq 2)$

Look up Normal curve, answer is .025

$$(\sigma = 10, n = 100), 1SE = \sigma/\sqrt{n} = 10/10 = 1$$

So,

$\Pr(\text{Test would have yielded a smaller difference from } H_0, H_0) = .97$

If didn't, it yielded  $Z > 2$

So there's evidence of departure from  $H_0$  (i.e.,  $\mu > 150$ )

And the .97 measures the well-testedness

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Keeping  $M = 152$  Consider different tests  
Test 2

1-sided normal testing

$H_0: \mu \leq 151$  vs.  $H_1: \mu > 151$  (Let  $\sigma = 10$ ,  $n = 100$ )

P-value associated with  $M = 152$

$Pr(M > 152; H_0: 151) = Pr(z > (152-151)/1) = Pr(Z > 1)$   
on standard Normal Curve: .16

$P\text{-value} = .16$ ;  $1 - p\text{ value} = .84$

$Pr(\text{Test would have yielded a smaller diff if } H_0 \text{ were true})$   
 $= .84$

*some indication*



Keeping  $M = 152$  Consider different tests

Test 3

1-sided normal testing

$H_0: \mu \leq 152$  vs.  $H_1: \mu > 152$  (Let  $\sigma = 10$ ,  $n = 100$ )

P-value associated with  $M = 152$

$Pr(M > 152; H_0: 152) = Pr(z > (152-152)/1) = Pr(Z > 0)$   
on standard Normal Curve: .50

$P\text{-value} = .5$ ;  $1 - p\text{ value} = .5$

$Pr(\text{Test would have yields a smaller diff if } H_0 \text{ were true}) = .5$

*Lousy evidence*

Go back to original test: 1-sided normal testing

$H_0: \mu \leq 150$  vs.  $H_1: \mu > 150$  (Let  $\sigma = 10$ ,  $n = 100$ )

*We don't want to infer merely there's some discrepancy from 150, we want to know the kind of magnitude information we saw*

*$M = 152$  is good evidence of  $\mu > 150$  SEV was .97*

*$M = 152$  is decent/some evidence of  $\mu > 151$  SEV was .84*

*$M = 152$  is lousy evidence of  $\mu > 152$  SEV was .5*

*Second third and 4<sup>th</sup> entries of p. 144.*